

# Econ 211

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## Expected Utility: The Classic Theory

# Motivating Example

- ▶ Suppose you are on the last round of the TV show *Who Wants to be a Millionaire*?
- ▶ You have narrowed down to two possible answers
  - ▶ Guess wrong: go home with \$32,000
  - ▶ Guess right: go home with \$1,000,000
- ▶ Walk away: go home with \$500,000 for certain
- ▶ What do you do?

# Gambles

- ▶ We need a way to make choices between uncertain options, eg gambles
- ▶ Consider a gamble called  $A$ , for example
  - ▶ Possible outcomes are indexed by  $i = 1, 2, 3, \dots, n$
  - ▶ Probability of outcome  $i$ :  $p_i$
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 $(\frac{1}{6}, \$1; \frac{1}{6}, \$2; \frac{1}{6}, \$3; \frac{1}{6}, \$4; \frac{1}{6}, \$5; \frac{1}{6}, \$6)$

# Expected Value

- ▶ Expected value of gamble  $A$ :

$$EV(A) = \sum_i^n p_i x_i = p_1 x_1 + p_2 x_2 + \dots + p_n x_n$$

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  - ▶ Guess from Millionaire:  $\frac{1}{2} \$1,000,000 + \frac{1}{2} \$32,000 = \$516,000$
  - ▶ Die roll:

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- ▶ Die roll:  $\frac{1}{6} \$1 + \frac{1}{6} \$2 + \frac{1}{6} \$3 + \frac{1}{6} \$4 + \frac{1}{6} \$5 + \frac{1}{6} \$6 = \$3.50$

# Expected Utility

- ▶ Expected utility
  - ▶ Consumer assigns utility  $u(x)$  to wealth  $x$
  - ▶ Expected utility theory says that

$$EU(A) = \sum_i^n p_i u(x_i) = p_1 u(x_1) + p_2 u(x_2) + \dots + p_n u(x_n)$$

- ▶ Consumers will choose the gamble that maximizes expected utility

# What Shape Should $u(x)$ Have?

- ▶ Consider the following game: I will flip a coin until the first heads comes up. If the first heads is on flip number  $n$ , then I'll pay you  $\$2^n$ . How much would you pay to play this game?
  - ▶ Originally proposed by Bernoulli (1738, reprinted 1954)
  - ▶ Known as the *St. Petersburg Paradox*
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  - ▶  $EV = \frac{1}{2}2 + \frac{1}{4}4 + \frac{1}{8}8 + \dots = 1 + 1 + 1 + \dots = \infty$
- ▶ It is clear that there is a *diminishing marginal utility of money*
  - ▶ Intuition: an extra \$1000 is massive windfall for a very poor person but not even noticeable for very rich person
- ▶ Means that  $u(x)$  is concave, which represents *risk-averse* preferences
  - ▶ Can also have *risk-seeking* preferences (convex  $u(x)$ ) or *risk-neutral* preference (linear  $u(x)$ )

# Risk Aversion

- ▶ One possible family of functions:  $u(x) = x^\alpha$
- ▶ Example:  $u(x) = \sqrt{x}$ , ie  $\alpha = \frac{1}{2}$ 
  - ▶ Expected utility of \$9 for certain?

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- ▶ Would decision-maker prefer \$9 for certain or a coin flip for \$25?  
certain amount, even though coin flip has expected payoff of  
\$12.50 > \$9.00



# Lab Evidence

- ▶ Subjects: 175 university students
- ▶ Choose either option A or B in *each* row:

TABLE 1—THE TEN PAIRED LOTTERY-CHOICE DECISIONS WITH LOW PAYOFFS

Option A	Option B	Expected payoff difference
1/10 of \$2.00, 9/10 of \$1.60	1/10 of \$3.85, 9/10 of \$0.10	\$1.17
2/10 of \$2.00, 8/10 of \$1.60	2/10 of \$3.85, 8/10 of \$0.10	\$0.83
3/10 of \$2.00, 7/10 of \$1.60	3/10 of \$3.85, 7/10 of \$0.10	\$0.50
4/10 of \$2.00, 6/10 of \$1.60	4/10 of \$3.85, 6/10 of \$0.10	\$0.16
5/10 of \$2.00, 5/10 of \$1.60	5/10 of \$3.85, 5/10 of \$0.10	−\$0.18
6/10 of \$2.00, 4/10 of \$1.60	6/10 of \$3.85, 4/10 of \$0.10	−\$0.51
7/10 of \$2.00, 3/10 of \$1.60	7/10 of \$3.85, 3/10 of \$0.10	−\$0.85
8/10 of \$2.00, 2/10 of \$1.60	8/10 of \$3.85, 2/10 of \$0.10	−\$1.18
9/10 of \$2.00, 1/10 of \$1.60	9/10 of \$3.85, 1/10 of \$0.10	−\$1.52
10/10 of \$2.00, 0/10 of \$1.60	10/10 of \$3.85, 0/10 of \$0.10	−\$1.85

- ▶ Repeated for 20x, 50x, 90x payoffs

Source: Holt and Laury (2002)

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- ▶ How should responses change with stakes?



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  3. Decreasing relative risk aversion: choices are less *risk* averse as stakes go up (i.e. switch earlier)

# Results: Holt and Laury

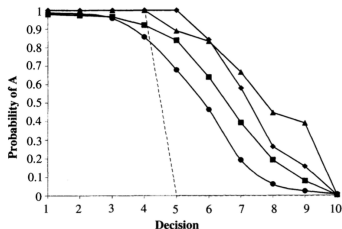


FIGURE 2. PROPORTION OF SAFE CHOICES IN EACH DECISION: DATA AVERAGES AND PREDICTIONS

*Note:* Data averages for low real payoffs [solid line with dots], 20x real [squares], 50x real [diamonds], 90x real payoffs [triangles], and risk-neutral prediction [dashed line].

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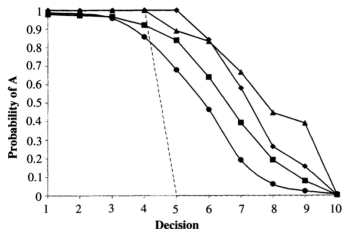


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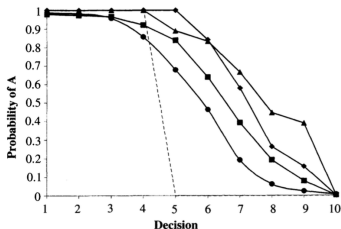


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  - ▶ Increasing relative risk aversion: note lines move out as stakes increase

Source: Holt and Laury (2002)

# Machina Triangles

- ▶ How do we graph risky prospects themselves?
- ▶ Suppose we fix payoff amounts  $x_1 < x_2 < x_3$
- ▶ Let  $p_1$ ,  $p_2$ , and  $p_3$  vary
- ▶ Since  $p_1 + p_2 + p_3 = 1$ , really just two degrees of freedom
- ▶ Put  $p_1$  on horizontal axis and  $p_3$  on vertical axis
- ▶ Possible gambles lie in the triangle defined by  $p_1 \geq 0$ ,  $p_3 \geq 0$ , and  $p_1 + p_3 \leq 1$ , hence the name *Machina triangle*

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  - ▶  $x_3$  for certain:  $(0, 1)$
  - ▶  $x_1$  and  $x_2$  with equal probability:

# Machina Triangles

- ▶ How do we graph risky prospects themselves?
- ▶ Suppose we fix payoff amounts  $x_1 < x_2 < x_3$
- ▶ Let  $p_1$ ,  $p_2$ , and  $p_3$  vary
- ▶ Since  $p_1 + p_2 + p_3 = 1$ , really just two degrees of freedom
- ▶ Put  $p_1$  on horizontal axis and  $p_3$  on vertical axis
- ▶ Possible gambles lie in the triangle defined by  $p_1 \geq 0$ ,  $p_3 \geq 0$ , and  $p_1 + p_3 \leq 1$ , hence the name *Machina triangle*
- ▶ Any gamble can be represented at a point on this graph:
  - ▶  $x_1$  for certain:  $(1, 0)$
  - ▶  $x_2$  for certain:  $(0, 0)$
  - ▶  $x_3$  for certain:  $(0, 1)$
  - ▶  $x_1$  and  $x_2$  with equal probability:  $(\frac{1}{2}, 0)$

# Machina Triangles

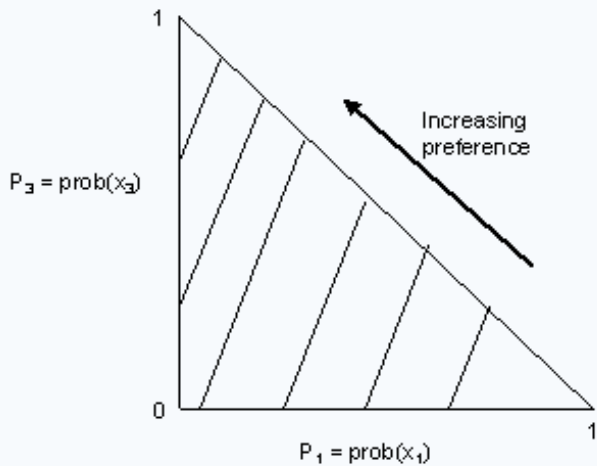
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# Machina Triangle



# Expected Utility in the Machina Triangle

- ▶ What do indifference curves in the Machina triangle look like for EUT?

# Expected Utility in the Machina Triangle

- ▶ What do indifference curves in the Machina triangle look like for EUT?
- ▶ Set  $EU = K$  :

$$p_1 u(x_1) + (1 - p_1 - p_3) u(x_2) + p_3 u(x_3) = K$$

- ▶ Solve for  $p_3$ :

$$p_3 = \frac{u(x_2) - u(x_1)}{u(x_3) - u(x_2)} p_1 + C$$

- ▶ Indifference curves on Machina triangle are straight parallel lines with positive slope (increasing preference up and to the left)
- ▶ More risk aversion: steeper slope

# Violations of Expected Utility Theory

# The Allais Paradox: Version 1

1. Choose your preferred option:

*A*: Receive \$100 million for certain

*B*: 10% chance of \$500 million, 89% chance of \$100 million, 1% chance of no money

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*A*: Receive \$100 million for certain

*B*: 10% chance of \$500 million, 89% chance of \$100 million, 1% chance of no money

2. Choose your preferred option:

*A'*: 11% chance of \$100 million, 89% chance of no money

*B'*: 10% chance of \$500 million, 90% chance of no money

# The Allais Paradox: Version 1

1. Choose your preferred option:

*A*: Receive \$100 million for certain

*B*: 10% chance of \$500 million, 89% chance of \$100 million, 1% chance of no money

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*A'*: 11% chance of \$100 million, 89% chance of no money

*B'*: 10% chance of \$500 million, 90% chance of no money

► Typical choice pattern?



# The Allais Paradox: Version 1

1. Choose your preferred option:

$A$ : Receive \$100 million for certain

$B$ : 10% chance of \$500 million, 89% chance of \$100 million, 1% chance of no money

2. Choose your preferred option:

$A'$ : 11% chance of \$100 million, 89% chance of no money

$B'$ : 10% chance of \$500 million, 90% chance of no money

► Typical choice pattern?  $A \succeq B$ ;  $B' \succeq A'$

►  $EU(A) =$

# The Allais Paradox: Version 1

1. Choose your preferred option:

A: Receive \$100 million for certain

B: 10% chance of \$500 million, 89% chance of \$100 million, 1% chance of no money

2. Choose your preferred option:

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B': 10% chance of \$500 million, 90% chance of no money

► Typical choice pattern?  $A \succeq B$ ;  $B' \succeq A'$

►  $EU(A) = u(100)$

►  $EU(B) =$

# The Allais Paradox: Version 1

1. Choose your preferred option:

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B': 10% chance of \$500 million, 90% chance of no money

► Typical choice pattern?  $A \succeq B$ ;  $B' \succeq A'$

►  $EU(A) = u(100)$

►  $EU(B) = .1u(500) + .89u(100) + .01u(0)$

►  $EU(A') =$

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►  $EU(B) = .1u(500) + .89u(100) + .01u(0)$

►  $EU(A') = .11u(100) + .89u(0)$

►  $EU(B') =$

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► Typical choice pattern?  $A \succeq B$ ;  $B' \succeq A'$

►  $EU(A) = u(100)$

►  $EU(B) = .1u(500) + .89u(100) + .01u(0)$

►  $EU(A') = .11u(100) + .89u(0)$

►  $EU(B') = .1u(500) + .9u(0)$

# Common Consequence Problem

- ▶ Suppose you choose  $A \succeq B$
- ▶ Then expected utility theory says you *must* choose  $A' \succeq B'$

$$EU(A') > EU(B')$$

$$\iff .11u(100) + .89u(0) \geq .1u(500) + .9u(0)$$

$$\iff .11u(100) + .89u(0) \geq .1u(500) + .89u(0) + .01u(0)$$

$$\iff .11u(100) + .89u(100) \geq .1u(500) + .89u(100) + .01u(0)$$

$$\iff u(100) \geq .1u(500) + .89u(100) + .01u(0)$$

$$\iff EU(A) > EU(B)$$

- ▶ Typical choice pattern is incompatible with expected utility theory
- ▶ Called *common consequence* version of the Allais Paradox, because I added the .89 chance of \$100 million to both sides

# The Allais Paradox: Version 2

1. Choose your preferred option:  
C: Receive \$100 million for certain  
D: 98% chance of \$500 million, 2% chance of no money
  2. Choose your preferred option:  
C': 1% chance of \$100 million, 99% chance of no money  
D': 0.98% chance of \$500 million, 99.02% chance of no money
- Typical choice pattern?

# The Allais Paradox: Version 2

1. Choose your preferred option:

$C$ : Receive \$100 million for certain

$D$ : 98% chance of \$500 million, 2% chance of no money

2. Choose your preferred option:

$C'$ : 1% chance of \$100 million, 99% chance of no money

$D'$ : 0.98% chance of \$500 million, 99.02% chance of no money

► Typical choice pattern?  $C \succeq D$ ;  $D' \succeq C'$

►  $EU(C) =$



# The Allais Paradox: Version 2

1. Choose your preferred option:

$C$ : Receive \$100 million for certain

$D$ : 98% chance of \$500 million, 2% chance of no money

2. Choose your preferred option:

$C'$ : 1% chance of \$100 million, 99% chance of no money

$D'$ : 0.98% chance of \$500 million, 99.02% chance of no money

► Typical choice pattern?  $C \succeq D$ ;  $D' \succeq C'$

►  $EU(C) = u(100)$

►  $EU(D) =$

# The Allais Paradox: Version 2

1. Choose your preferred option:

$C$ : Receive \$100 million for certain

$D$ : 98% chance of \$500 million, 2% chance of no money

2. Choose your preferred option:

$C'$ : 1% chance of \$100 million, 99% chance of no money

$D'$ : 0.98% chance of \$500 million, 99.02% chance of no money

► Typical choice pattern?  $C \succeq D$ ;  $D' \succeq C'$

►  $EU(C) = u(100)$

►  $EU(D) = .98u(500) + .02u(0)$

►  $EU(C') =$

# The Allais Paradox: Version 2

1. Choose your preferred option:

$C$ : Receive \$100 million for certain

$D$ : 98% chance of \$500 million, 2% chance of no money

2. Choose your preferred option:

$C'$ : 1% chance of \$100 million, 99% chance of no money

$D'$ : 0.98% chance of \$500 million, 99.02% chance of no money

► Typical choice pattern?  $C \succeq D$ ;  $D' \succeq C'$

►  $EU(C) = u(100)$

►  $EU(D) = .98u(500) + .02u(0)$

►  $EU(C') = .01u(100) + .99u(0)$

►  $EU(D') =$

# The Allais Paradox: Version 2

1. Choose your preferred option:

$C$ : Receive \$100 million for certain

$D$ : 98% chance of \$500 million, 2% chance of no money

2. Choose your preferred option:

$C'$ : 1% chance of \$100 million, 99% chance of no money

$D'$ : 0.98% chance of \$500 million, 99.02% chance of no money

► Typical choice pattern?  $C \succeq D$ ;  $D' \succeq C'$

►  $EU(C) = u(100)$

►  $EU(D) = .98u(500) + .02u(0)$

►  $EU(C') = .01u(100) + .99u(0)$

►  $EU(D') = .0098u(500) + .9902u(0)$

# Common Ratio Problem

- ▶ Suppose we observe  $C \succeq D$
- ▶ Then expected utility theory says we *must* have  $C' \succeq D'$

$$EU(C) > EU(D)$$

$$\iff u(100) \geq .98u(500) + .02u(0)$$

$$\iff 0.01u(100) \geq .0098u(500) + .0002u(0)$$

$$\iff 0.01u(100) + 0.99u(0) \geq .0098u(500) + .0002u(0) + 0.99u(0)$$

$$\iff 0.01u(100) + 0.99u(0) \geq .0098u(500) + .9902u(0)$$

$$\iff EU(C') > EU(D')$$

- ▶ Called *common ratio* version of the Allais Paradox, because I multiplied both sides of the equation by 0.01

# What Is Going On?

- ▶ Expected utility theory says we should have  $A \succeq B \iff A' \succeq B'$  and  $C \succeq D \iff C' \succeq D'$
- ▶ So if actual behavior doesn't follow these results, expected utility theory must not represent people's true preferences?
- ▶ Next time we will see a theory that does explain these choice patterns better