

### Motivation

- ▶ Which would you rather have?
  - ▶ \$100 today OR \$95 one month
  - ▶ \$100 today OR \$97 one month
  - ▶ \$100 today OR \$99 one month
  - ▶ \$100 today OR \$101 one month
  - ▶ \$100 today OR \$103 one month
  - ▶ \$100 today OR \$105 one month
- ▶ If you value money today more than the same amount of money in the future, then we say you are *impatient*

### Consumption Over Time

- ▶ Stream of consumption (or wealth or income) over  $T$  time periods, starting with period 1:

$$c = (c_1, c_2, c_3, \dots, c_T)$$

- ▶ Example:  $T = 3$  periods:  $(c_1, c_2, c_3) = (\$5, \$10, \$0)$
- ▶ Utility is function of the entire stream of income:

$$U(c) = f(c_1, c_2, c_3, \dots, c_T)$$

- ▶ If *impatient*, then would prefer to have an extra dollar today rather than tomorrow, implying

$$\frac{\partial U}{\partial c_t} > \frac{\partial U}{\partial c_{t+1}}$$

or equivalently:

$$\frac{\frac{\partial U}{\partial c_{t+1}}}{\frac{\partial U}{\partial c_t}} < 1$$

## Time Consistency

- ▶ Suppose decision maker (DM) is making plan for consumption in future (possibly uncertain) states
- ▶ In the standard model, they make a complete contingent plan and stick to it
  - ▶ That is, they are happy to commit to their plan at any earlier date
  - ▶ When they arrive at the future state, they will not want to change their plan
  - ▶ They are *time consistent*
- ▶ A formal definition
  - ▶ Let consumption for period  $\tau$  chosen at period  $t \leq \tau$  be  $c(\tau|t)$
  - ▶ DM is time consistent if  $c(\tau|t) = c(\tau|\tau)$  for any  $t \leq \tau$

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## The General Discounting Model

- ▶ Consider a stream of consumption over time, starting with period  $t$ :  $c_t^T = (c_t, c_{t+1}, c_{t+2}, \dots, c_T)$
- ▶ Discounted utility model says that the utility at period  $t$  of the whole stream is

$$U_t(c_t^T) = D(0)u(c_t) + D(1)u(c_{t+1}) + D(2)u(c_{t+2}) + \dots + D(T-t)u(c_T)$$

$$= \sum_{\tau=t}^T D(\tau-t)u(c_\tau)$$

- ▶ Impatience implies that  $D(t+1) \leq D(t)$ , ie  $D$  is decreasing
  - ▶ Same amount of consumption has smaller impact on utility if it is farther in the future
- ▶ Typically set  $D(0) = 1$

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## MRS Between Periods

- ▶ Consider two periods  $t+k$  and  $t+k+1$
- ▶ What is MRS of consumption between these two periods?
  - ▶  $\frac{dU_t}{dc_{t+k}} = D(k)u'(c_{t+k})$
  - ▶  $\frac{dU_t}{dc_{t+k+1}} = D(k+1)u'(c_{t+k+1})$
  - ▶  $MRS = \frac{D(k+1)u'(c_{t+k+1})}{D(k)u'(c_{t+k})}$
- ▶ If we assume price of consumption is the same in all periods, then we have
  - ▶  $MRS = 1 \implies \frac{D(k+1)}{D(k)} = \frac{u'(c_{t+k})}{u'(c_{t+k+1})}$

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## Time Consistency and Discounting

- ▶ Suppose DM is in period  $t$ , making decision about consumption in period  $r$  and  $r+1$  in the future
  - ▶ Tradeoff will be governed by  $\frac{D(r-t)}{D(r+1-t)} = \frac{u'(c_r)}{u'(c_{r+1})}$
- ▶ Suppose DM is in period  $s > t$ , making decision about consumption in period  $r$  and  $r+1$  (still in the future)
  - ▶ Tradeoff will be governed by  $\frac{D(r-s)}{D(r+1-s)} = \frac{u'(c_r)}{u'(c_{r+1})}$
- ▶ Time consistency says optimal  $c_r$  and  $c_{r+1}$  should not depend on whether consumption decision is made in period  $s$  or  $t$
- ▶ Therefore  $\frac{D(r-s)}{D(r+1-s)} = \frac{D(r-t)}{D(r+1-t)}$  for any  $r, s, t$
- ▶ For time consistency, any discount factors separated by same amount of periods should have the same ratio

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## Discounted Expected Utility

- ▶ In particular, if periods are consecutive, we must have for any  $k$

$$\frac{D(k+1)}{D(k)} = \frac{D(1)}{D(0)} = \delta$$

- ▶  $\delta$  is the *discount factor*
- ▶ Then  $D(k) = \delta^k$ , where  $0 \leq \delta \leq 1$
- ▶ Thus time-consistency implies that we can write utility as

$$\begin{aligned} U_t(c_t^T) &= u(c_t) + \delta u(c_{t+1}) + \delta^2 u(c_{t+2}) + \dots + \delta^{T-t} u(c_T) \\ &= \sum_{\tau=t}^T \delta^{(\tau-t)} u(c_\tau) \end{aligned}$$

- ▶ This is *geometric* or *exponential* discounting
- ▶ Agent becomes more impatient as  $\delta \rightarrow 0$

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## Example: Doing Your Laundry

- ▶ Suppose your utility each day is proportional to how many clean outfits you have to wear
- ▶ On Friday that you have just 2 clean outfits left
- ▶ You can do laundry on Friday, Saturday, or Sunday, or Monday
- ▶ Doing laundry is annoying:  $-5$  utils the day you choose to do it
- ▶ Doing laundry gets you 5 clean outfits, but you use one each day
- ▶ In summary:

	Utility on day			
	F	Sa	Su	M
Do laundry Fri	-3	5	4	3
Do laundry Sat	2	-4	5	4
Do laundry Sun	2	1	-5	5
Do laundry Mon	2	1	0	-5

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## When Do You Do Your Laundry?

- ▶ From Friday's perspective, what is overall utility of doing laundry on

Friday?

Saturday?

Sunday?

Monday?

- ▶ Utilities under various values of  $\delta$ :

		Total utility if $\delta =$			
		1	0.6	0.52	0.25
Do laundry Fri	9*	2.09	1.10	-1.45	
Do laundry Sat	7	2.27*	1.83	1.38	
Do laundry Sun	3	1.88	1.87*	2.02	
Do laundry Mon	-2	1.52	1.82	2.17*	

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## Checking Follow Through

- ▶ Suppose your  $\delta = 0.6$ , so on Friday you decide to do laundry on Saturday
- ▶ Saturday morning comes, and you re-evaluate your choices
- ▶ Note that "today", ie period 1, is now Saturday
- ▶ From Saturday's perspective, what is utility of doing laundry on

Saturday?

Sunday?

Monday?

- ▶ Will you follow through with plan?

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## How Do We Measure Time Preferences?

- ▶ Suppose you are indifferent between \$100 today and \$X in one month
- ▶ Utility of \$100 today:  $u(\$100)$
- ▶ Utility of \$X next month:  $\delta u(\$X)$  (assuming monthly discount factor)
- ▶ Thus we must have  $u(\$100) = \delta u(\$X)$ , which implies

$$\delta = \frac{u(\$100)}{u(\$X)}$$

- ▶ If we make the assumption that  $u(x) = x$ , then

$$\delta = \frac{100}{X}$$

- ▶ Thus we can estimate time preferences by looking at switch point on price list

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## Results

- ▶ What behavior do we expect from discounted exponential utility model?
- ▶ What actually happened?
  - ▶ Treatment 1 (immediate):
  - ▶ Treatment 2 (delay):

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## Lab Evidence: McClure et al (2007)

- ▶ Subjects told to come into the lab thirsty
- ▶ Experiment lasts at least 30 minutes
- ▶ Treatment 1 (immediate): choose either
  - ▶ 1 juice now (early) OR
  - ▶ 2 juices in 5 minutes (later)
- ▶ Treatment 2 (delay): choose either
  - ▶ 1 juice in 20 minutes (early) OR
  - ▶ 2 juices in 25 minutes (later)
- ▶ Subjects know this is their only chance to get a drink during the experiment

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## Field Evidence: Read, Loewenstein, and Kalyanaraman (1999)

- ▶ Subjects get vouchers from certain movies off of a list
- ▶ List includes “high brow” and “low brow” movies
  - ▶ “High brow” movies: Schindler’s List, Like Water for Chocolate
  - ▶ “Low brow” movies: The Mask, Mrs. Doubtfire
- ▶ Treatment 1 (immediate): Subjects pick movie for tonight
- ▶ Treatment 2 (delay): Subjects pick movie for one week from now
- ▶ Expect results from discounted exponential model?
- ▶ Results:
  - ▶ Treatment 1 (immediate):
  - ▶ Treatment 2 (delay):

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## Time Inconsistency

- ▶ In actuality, we observe much behavior that is *time inconsistent*
  - ▶ That is, consumers make a different choice for tomorrow's consumption when asked today vs when asked tomorrow
  - ▶ Such consumers will have a *self-control problem*
- ▶ Also, we see that some people are aware of their time inconsistency
  - ▶ A *naïve* agent believes (incorrectly) that he will follow through on his plans
  - ▶ A *sophisticated* agent knows that she may not follow through, so she may look for ways to *commit* herself to the plan

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## Quasi-hyperbolic Discounting

- ▶ First proposed by Strotz (1955) and popularized by Laibson (1997)
- ▶ Specifies discount factor for  $k > 0$  as

$$D(k) = \beta \delta^k$$

where  $0 \leq \beta \leq 1$

- ▶ Note  $D(0) = 1$  still (and this is important!)
- ▶ Plugging in to utility function:

$$U_t = u(c_t) + \beta [\delta u(c_{t+1}) + \delta^2 u(c_{t+2}) + \delta^3 u(c_{t+3}) + \dots]$$

- ▶ Also known as  $\beta$ - $\delta$  *discounting* or *present-bias*

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## Comparing Geometric and Quasi-hyperbolic Discounting

Ratio	Geometric	Hyberbolic
$\frac{D(1)}{D(0)}$	$\delta$	$\beta \delta$
$\frac{D(k+1)}{D(k)}$ for $k > 0$	$\delta$	$\delta$

- ▶ Any case where  $\frac{D(k+1)}{D(k)}$  depends on  $k$  will in general lead to time inconsistent behavior
- ▶ It is the  $\beta$  in the  $\beta$ - $\delta$  model that is making behavior time-inconsistent

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## Example: How QHD Leads to Time-Inconsistency

- ▶ Three periods:  $t = 0, 1, 2$
- ▶ Two options:
  1. Eat well:  $u_1 = 5, u_2 = 10$
  2. Eat poorly:  $u_1 = 8, u_2 = 6$
- ▶ Assume that DM has QHD preferences with  $\beta = \frac{1}{2}, \delta = 1$
- ▶ Decision in period 0:
- ▶ Decision in period 1:

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