

Econ 311: Behavioral and Experimental Economics

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Introduction to Time Preferences

Motivation

- ▶ Which would you rather have?
 - ▶ \$100 today OR \$95 one month

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 - ▶ \$100 today OR \$105 one month
- ▶ If you value money today more than the same amount of money in the future, then we say you are *impatient*

Consumption Over Time

- ▶ Stream of consumption (or wealth or income) over T time periods, starting with period 1:

$$c = (c_1, c_2, c_3, \dots, c_T)$$

- ▶ Example: $T = 3$ periods: $(c_1, c_2, c_3) = (\$5, \$10, \$0)$

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- ▶ If *impatient*, then would prefer to have an extra dollar today rather than tomorrow, implying

$$\frac{\partial U}{\partial c_t} > \frac{\partial U}{\partial c_{t+1}}$$

or equivalently:

$$\frac{\frac{\partial U}{\partial c_{t+1}}}{\frac{\partial U}{\partial c_t}} < 1$$

Time Consistency

- ▶ Suppose decision maker (DM) is making plan for consumption in future (possibly uncertain) states
- ▶ In the standard model, they make a complete contingent plan and stick to it
 - ▶ That is, they are happy to commit to their plan at any earlier date
 - ▶ When they arrive at the future state, they will not want to change their plan
 - ▶ They are *time consistent*

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 - ▶ When they arrive at the future state, they will not want to change their plan
 - ▶ They are *time consistent*
- ▶ A formal definition
 - ▶ Let consumption for period τ chosen at period $t \leq \tau$ be $c(\tau|t)$
 - ▶ DM is time consistent if $c(\tau|t) = c(\tau|\tau)$ for any $t \leq \tau$

The General Discounting Model

- ▶ Consider a stream of consumption over time, starting with period t :
 $c_t^T = (c_t, c_{t+1}, c_{t+2}, \dots, c_T)$
- ▶ Discounted utility model says that the utility at period t of the whole stream is

$$\begin{aligned} U_t(c_t^T) &= D(0)u(c_t) + D(1)u(c_{t+1}) + D(2)u(c_{t+2}) + \dots + D(T-t)u(c_T) \\ &= \sum_{\tau=t}^T D(\tau-t)u(c_\tau) \end{aligned}$$

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- ▶ Impatience implies that $D(t+1) \leq D(t)$, ie D is decreasing
 - ▶ Same amount of consumption has smaller impact on utility if it is farther in the future
- ▶ Typically set $D(0) = 1$

MRS Between Periods

- ▶ Consider two periods $t + k$ and $t + k + 1$
- ▶ What is MRS of consumption between these two periods?
 - ▶ $\frac{dU_t}{dc_{t+k}} = D(k)u'(c_{t+k})$
 - ▶ $\frac{dU_t}{dc_{t+k+1}} = D(k+1)u'(c_{t+k+1})$
 - ▶ $MRS = \frac{D(k+1)u'(c_{t+k+1})}{D(k)u'(c_{t+k})}$
- ▶ If we assume price of consumption is the same in all periods, then we have
 - ▶ $MRS = 1 \implies \frac{D(k+1)}{D(k)} = \frac{u'(c_{t+k})}{u'(c_{t+k+1})}$

Time Consistency and Discounting

- ▶ Suppose DM is in period t , making decision about consumption in period r and $r + 1$ in the future
 - ▶ Tradeoff will be governed by $\frac{D(r-t)}{D(r+1-t)} = \frac{u'(c_r)}{u'(c_{r+1})}$

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- ▶ Therefore $\frac{D(r-s)}{D(r+1-s)} = \frac{D(r-t)}{D(r+1-t)}$ for any r, s, t
- ▶ For time consistency, any discount factors separated by same amount of periods should have the same ratio

Discounted Expected Utility

- ▶ In particular, if periods are consecutive, we must have for any k

$$\frac{D(k+1)}{D(k)} = \frac{D(1)}{D(0)} = \delta$$

- ▶ δ is the *discount factor*

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- ▶ δ is the *discount factor*
- ▶ Then $D(k) = \delta^k$, where $0 \leq \delta \leq 1$
- ▶ Thus time-consistency implies that we can write utility as

$$\begin{aligned} U_t(c_t^T) &= u(c_t) + \delta u(c_{t+1}) + \delta^2 u(c_{t+2}) + \dots + \delta^{T-t} u(c_T) \\ &= \sum_{\tau=t}^T \delta^{(t-\tau)} u(c_\tau) \end{aligned}$$

- ▶ This is *geometric* or *exponential* discounting
- ▶ Agent becomes more impatient as $\delta \rightarrow 0$

Example: Doing Your Laundry

- ▶ Suppose your utility each day is proportional to how many clean outfits you have to wear
- ▶ On Friday that you have just 2 clean outfits left
- ▶ You can do laundry on Friday, Saturday, or Sunday, or Monday
- ▶ Doing laundry is annoying: -5 utils the day you choose to do it
- ▶ Doing laundry gets you 5 clean outfits, but you use one each day
- ▶ In summary:

	Utility on day			
	F	Sa	Su	M
Do laundry Fri	-3	5	4	3
Do laundry Sat	2	-4	5	4
Do laundry Sun	2	1	-5	5
Do laundry Mon	2	1	0	-5

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Saturday?	2	$+\delta(-4)$	$+\delta^2(5)$	$+\delta^3(4)$
Sunday?	2	$+\delta(1)$	$+\delta^2(-5)$	$+\delta^3(5)$
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- Utilities under various values of δ :

	Total utility if $\delta =$			
	1	0.6	0.52	0.25
Do laundry Fri	9*	2.09	1.10	-1.45
Do laundry Sat	7	2.27*	1.83	1.38
Do laundry Sun	3	1.88	1.87*	2.02
Do laundry Mon	-2	1.52	1.82	2.17*

Checking Follow Through

- ▶ Suppose your $\delta = 0.6$, so on Friday you decide to do laundry on Saturday
- ▶ Saturday morning comes, and you re-evaluate your choices
- ▶ Note that “today”, ie period 1, is now Saturday
- ▶ From Saturday’s perspective, what is utility of doing laundry on

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Saturday?	-4	$+(0.6)(5)$	$+(0.6)^2(4)$	$= 0.44$
Sunday?	1	$+(0.6)(-5)$	$+(0.6)^2(5)$	$= -0.20$
Monday?	1	$+(0.6)(0)$	$+(0.6)^2(-5)$	$= -0.80$

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- ▶ Will you follow through with plan?

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- ▶ Will you follow through with plan? Yes, since utility of doing on Saturday is still highest

How Do We Measure Time Preferences?

- ▶ Suppose you are indifferent between \$100 today and \$X in one month
- ▶ Utility of \$100 today: $u(\$100)$
- ▶ Utility of \$X next month: $\delta u(\$X)$ (assuming monthly discount factor)
- ▶ Thus we must have $u(\$100) = \delta u(\$X)$, which implies

$$\delta = \frac{u(\$100)}{u(\$X)}$$

- ▶ If we make the assumption that $u(x) = x$, then

$$\delta = \frac{100}{X}$$

- ▶ Thus we can estimate time preferences by looking at switch point on price list

Lab Evidence: McClure et al (2007)

- ▶ Subjects told to come into the lab thirsty
- ▶ Experiment lasts at least 30 minutes
- ▶ Treatment 1 (immediate): choose either
 - ▶ 1 juice now (early) OR
 - ▶ 2 juices in 5 minutes (later)

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 - ▶ 1 juice now (early) OR
 - ▶ 2 juices in 5 minutes (later)
- ▶ Treatment 2 (delay): choose either
 - ▶ 1 juice in 20 minutes (early) OR
 - ▶ 2 juices in 25 minutes (later)
- ▶ Subjects know this is their only chance to get a drink during the experiment

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 - ▶ Reduces to $u(1) > \delta u(2)$, same as Treatment 1
 - ▶ Thus we expect same percentage subjects choosing early option in both treatments

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 - ▶ Treatment 1 (immediate): 60% choose early option
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 - ▶ Reduces to $u(1) > \delta u(2)$, same as Treatment 1
 - ▶ Thus we expect same percentage subjects choosing early option in both treatments
- ▶ What actually happened?
 - ▶ Treatment 1 (immediate): 60% choose early option
 - ▶ Treatment 2 (delay): 30% choose early option

Field Evidence: Read, Loewenstein, and Kalyanaraman (1999)

- ▶ Subjects get vouchers from certain movies off of a list
- ▶ List includes “high brow” and “low brow” movies
 - ▶ “High brow” movies: Schindler’s List, Like Water for Chocolate
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- ▶ Expect results from discounted exponential model?
 - ▶ As in previous experiment, expect same percentage choosing low brow movie in two treatments
- ▶ Results:

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- ▶ Expect results from discounted exponential model?
 - ▶ As in previous experiment, expect same percentage choosing low brow movie in two treatments
- ▶ Results:
 - ▶ Treatment 1 (immediate): 66% low brow

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 - ▶ Treatment 1 (immediate): 66% low brow
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- ▶ Treatment 1 (immediate): Subjects pick movie for tonight
- ▶ Treatment 2 (delay): Subjects pick movie for one week from now
- ▶ Expect results from discounted exponential model?
 - ▶ As in previous experiment, expect same percentage choosing low brow movie in two treatments
- ▶ Results:
 - ▶ Treatment 1 (immediate): 66% low brow
 - ▶ Treatment 2 (delay): 37% low brow

Time Inconsistency

- ▶ In actuality, we observe much behavior that is *time inconsistent*
 - ▶ That is, consumers make a different choice for tomorrow's consumption when asked today vs when asked tomorrow
 - ▶ Such consumers will have a *self-control problem*

Time Inconsistency

- ▶ In actuality, we observe much behavior that is *time inconsistent*
 - ▶ That is, consumers make a different choice for tomorrow's consumption when asked today vs when asked tomorrow
 - ▶ Such consumers will have a *self-control problem*
- ▶ Also, we see that some people are aware of their time inconsistency
 - ▶ A *naive* agent believes (incorrectly) that he will follow through on his plans
 - ▶ A *sophisticated* agent knows that she may not follow through, so she may look for ways to *commit* herself to the plan

Quasi-hyperbolic Discounting

- ▶ First proposed by Strotz (1955) and popularized by Laibson (1997)
- ▶ Specifies discount factor for $k > 0$ as

$$D(k) = \beta \delta^k$$

where $0 \leq \beta \leq 1$

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- ▶ Plugging in to utility function:

$$U_t = u(c_t) + \beta [\delta u(c_{t+1}) + \delta^2 u(c_{t+2}) + \delta^3 u(c_{t+3}) + \dots]$$

- ▶ Also known as β - δ *discounting* or *present-bias*

Comparing Geometric and Quasi-hyperbolic Discounting

Ratio	Geometric	Hyperbolic
$\frac{D(1)}{D(0)}$	δ	$\beta\delta$
$\frac{D(k+1)}{D(k)}$ for $k > 0$	δ	δ

- ▶ Any case where $\frac{D(k+1)}{D(k)}$ depends on k will in general lead to time inconsistent behavior
- ▶ It is the β in the β - δ model that is making behavior time-inconsistent

Example: How QHD Leads to Time-Inconsistency

- ▶ Three periods: $t = 0, 1, 2$
- ▶ Two options:
 1. Eat well: $u_1 = 5, u_2 = 10$
 2. Eat poorly: $u_1 = 8, u_2 = 6$
- ▶ Assume that DM has QHD preferences with $\beta = \frac{1}{2}, \delta = 1$

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- ▶ Assume that DM has QHD preferences with $\beta = \frac{1}{2}, \delta = 1$
- ▶ Decision in period 0:
 - ▶ Eat well: $U = \frac{1}{2}5 + \frac{1}{2}10 = 7.5$
 - ▶ Eat poorly: $U = \frac{1}{2}8 + \frac{1}{2}6 = 7$
 - ▶ Decision: eat well

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- ▶ Decision in period 1:

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- ▶ Assume that DM has QHD preferences with $\beta = \frac{1}{2}, \delta = 1$
- ▶ Decision in period 0:
 - ▶ Eat well: $U = \frac{1}{2}5 + \frac{1}{2}10 = 7.5$
 - ▶ Eat poorly: $U = \frac{1}{2}8 + \frac{1}{2}6 = 7$
 - ▶ Decision: eat well
- ▶ Decision in period 1:
 - ▶ Eat well: $U = 5 + \frac{1}{2}10 = 10$
 - ▶ Eat poorly: $U = 8 + \frac{1}{1}6 = 11$
 - ▶ Decision: eat poorly