

What is an Auction?

- ▶ An auction is a way of allocating a single good to one of many interested buyers
- ▶ Buyers typically submit *bids* which can be any positive number
- ▶ Who gets the item, and how much they pay for it, is a function of the all the bids
- ▶ Note that auctions are a type of game:
 - ▶ Bidders are the players
 - ▶ Bids are the strategies
 - ▶ Payoffs are winnings minus payments

When Are Auctions Used?

- ▶ Thin markets: unique/rare items such as art, collectibles
- ▶ Thick markets: common/mundane items such as fish, livestock, flowers, online ads
- ▶ Procurement: government seeking private companies to complete large projects
- ▶ Resources: government selling rights oil, land, bandwidth, pollution

Why Are Auctions Used so Frequently?

- ▶ Why do sellers like auctions?
- ▶ Why do buyers like auctions?
- ▶ Why do economists like auctions?

32 / 43

Private vs Common Value

- ▶ Typically assume good being auctioned off has either *private value* or *common value*
 - ▶ Private value
 - ▶ Each bidder has different valuation of the good
 - ▶ Players do not know the values of others
 - ▶ Example?
 - ▶ Common value
 - ▶ Each bidder has same value of good
 - ▶ Before submitting bid, bidders get a noisy signal of the value
 - ▶ Only find out the exact value after the good is allocated
 - ▶ Example?

33 / 43

Different Ways to Run an Auction

- ▶ Ascending bid English auction: Bidders call out increasing prices until only one bidder is left
- ▶ Sealed bid auction: Bidders write down a bid, seal in envelope, and turn in to auctioneer
 - ▶ Winner is person who submits highest bid
 - ▶ Pays either their bid (*first price*) or the next-highest bid (*second price*)
- ▶ Descending bid Dutch auction: start with high price, lower until one person is willing to buy at that price
 - ▶ Called Dutch auction because it's how flowers are sold in Amsterdam

34 / 43

Optimal in Private Values English Ascending-Bid Auction

- ▶ Suppose you are in an English auction
- ▶ Strategy: point at which you will drop out of auction (call this your bid)
- ▶ Is bid above your valuation a good idea?
- ▶ Is bid below your valuation a good idea?

35 / 43

Optimal Behavior in Second-Price Sealed Bid Auction

- ▶ Suppose you are in a private values second price sealed bid auction
- ▶ Is bidding above your valuation a good idea?
- ▶ Is bidding below your valuation a good idea?
- ▶ Note that English auction and second-price sealed bid are *strategic equivalents*: they have same optimal strategy

36 / 43

Optimal Behavior in First-Price Sealed Bid Auction

- ▶ Suppose you are in a *first-price* sealed bid auction where bidders have private values
 - ▶ The price the winner will pay is their own bid
- ▶ Is bidding above your valuation a good idea?
- ▶ Is bidding exactly your valuation a good idea?

37 / 43

Formal Analysis for First-Price Sealed Bid

- ▶ Suppose you are bidding against a computer which will bid randomly between \$0 and \$10
- ▶ Your valuation: $v \in [\$0, \$10]$
- ▶ Your bid: b
- ▶ What is optimal bid b^* you should make?
 - ▶ Your payoff if you win with bid b :
 - ▶ Probability that you win with this bid:
 - ▶ Expected payoff (assume you are risk neutral):
 - ▶ First order condition with respect to b :
 - ▶ Solve to find $b^* =$
- ▶ In general, if N people bidding, best response bid is $b^* =$

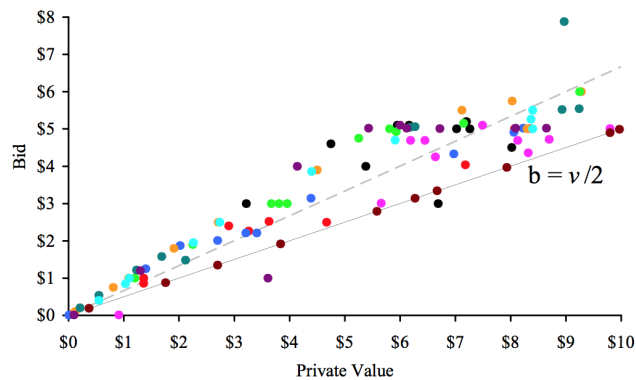
38 / 43

Formal Analysis, cont

- ▶ Now assume you are bidding against another player instead of computer
- ▶ Is both players bidding half their valuation a NE?
 - ▶ Suppose player $i = 1, 2$ has value $v_i \sim U[0, 10]$ and enters bid b_i
 - ▶ Suppose player 2 is indeed bidding $\frac{v_2}{2}$
 - ▶ Player 1's probability of winning is
- ▶ Expected payoff for 1 is then
- ▶ Can check this payoff is maximized at

39 / 43

Actual Behavior in Private Values First Price Auction



- ▶ Best fit line has slope of about $\frac{2}{3}$ (predicted should be $\frac{1}{2}$)

Source: Holt

40 / 43

Optimal Strategy in Private Values Dutch Auction

- ▶ Recall that in Dutch auction, value starts very high and drops until someone stops it
- ▶ Makes sense to pick a point at which you will stop the auction (if someone has not stopped it already)
- ▶ Does it make sense to have stopping point above your value?
- ▶ Does it make sense to have stopping point exactly at your value?

41 / 43

How do People Actually Bid in Auctions?

- ▶ In summary, we found:
 - ▶ In first-price sealed bid and descending clock, bidders should shade their bid (by $\frac{N-1}{N}$)
 - ▶ In second-price sealed bid and English, bidders should bid their valuation (ie no shading of bid)
- ▶ Do people actually play this way? Let's look at the data from our class exercise

42 / 43

Common Value Auctions

- ▶ Now move to common value auctions
 - ▶ Item is worth the same to everyone
 - ▶ Get a noisy signal of valuation before you buy
 - ▶ Find out exact value only after winner is announced
- ▶ Example: oil lease
 - ▶ Suppose gov't holds land that is worth v to gov't
 - ▶ Gov't decides to auction off land to oil companies
 - ▶ Companies can extract more value from land, say $1.5v$
 - ▶ Companies don't know v exactly; only get signal $v + \epsilon$
 - ▶ Winner is company that submit highest bid
 - ▶ This will be company that gets most optimistic signal
 - ▶ Winner company almost certainly will bid more than land is worth
 - ▶ This is called *winner's curse*

43 / 43