

Econ 301: Microeconomic Analysis

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Course Overview

What This Course is About

- ▶ Tools that economists use to analyze behavior
 - ▶ Calculus
 - ▶ Optimization
 - ▶ Equilibrium
 - ▶ Game theory
- ▶ Applications to that economists typically study
 - ▶ Consumers
 - ▶ Producers
 - ▶ Governments
 - ▶ Markets
 - ▶ Small groups

Consumers

- ▶ How do people choose between options? *Preferences, choices*
- ▶ How do we summarize their preferences with a convenient mathematical function? *Utility*
- ▶ How do people deal with limited options? *Budget constraints*
- ▶ How do people make choices when uncertainty is involved?
Expected utility, asset markets
- ▶ How do people make choices over several time periods?
Intertemporal choice, price indexes
- ▶ How do these choices affect entire markets? *Consumer surplus, market demand*

Producers

- ▶ How do firms decide how much to produce? *Technology, cost minimization, cost curves*
- ▶ How do these choices affect the entire market? *Firm supply, industry supply*
- ▶ What happens if you don't have a competitive market? *Monopoly, oligopoly*

Interactions

- ▶ How do supply and demand interact? *Equilibrium*
- ▶ What are the side effects of markets? *Externalities*
- ▶ What happens if small groups interact rather than large markets?
Game theory, game applications
- ▶ What happens if one side of the market knows more than the other? *Asymmetric information*

Thinking like an Economist

1. Start with a model

- ▶ Agent or agents
- ▶ Their objectives
- ▶ Options available to them

2. Solve the model

- ▶ Optimize behavior of agents, usually with calculus
- ▶ If agents interact, find equilibrium

3. Poke the model

- ▶ What happens if constraints change? *Comparative statics*
- ▶ Could agents be made better off? *Welfare, efficiency*

Who You Are

- ▶ Econ majors and minors
- ▶ Should have already taken Econ 300
- ▶ Introduce yourselves
 - ▶ First name
 - ▶ Major(s)
 - ▶ Where you're from
 - ▶ Tell us something we don't know

Administrative Details

- ▶ Syllabus overview
- ▶ Moodle overview

Mindset

- ▶ May not get everything in lecture the first time
 - ▶ Review your notes after lecture
 - ▶ Rework examples from slides
- ▶ Iterate to success:
 - ▶ Work by yourself
 - ▶ Work with study group
 - ▶ Go to TA session
 - ▶ Go to office hours
 - ▶ Post to Piazza
 - ▶ Repeat as needed
- ▶ Many PS and exam problems will require a translation or wrinkle of some kind

Advice from Former Students

- ▶ “Work hard. Attend classes, pay attention, and interact actively.”
- ▶ “Go to every class, and go to extra help sessions whenever possible.”
- ▶ “Study the material week by week, not just before a big test. Knowledge is cumulative in this class.”
- ▶ “[M]ake a study group to collaborate on problem sets and test review.”
- ▶ “Go to the TA. [...] Work through as many problems as you can get your hands on. Don't be afraid to ask questions.”
- ▶ “Do not be afraid to answer questions when he randomly calls on you, he does not bite.”

Mathematics Review

Mathematics in This Course

- ▶ Because this course is about microeconomic ***theory***, deal often with abstract equations and quantities
- ▶ Use lots of algebra and calculus
- ▶ For reasons outside my control, we will do some multivariate calculus even though this is not a pre-req
- ▶ If this is new to you, ask for help from me/TAs/classmates/math workshop

Algebra Tricks

- ▶ Exponents:

$$x^a x^b = x^{(a+b)}$$

$$\frac{x^a}{x^b} = x^a x^{-b} = x^{(a-b)}$$

$$(x^m)^n = x^{mn}$$

- ▶ Logarithms:

$$\log x^b = b \log x$$

$$\log(xy) = \log x + \log y$$

$$\log\left(\frac{x}{y}\right) = \log x - \log y$$

Basic Derivatives

	Function	Derivative
Derivative of a constant	$f(x) = a$	$\frac{d}{dx} f(x) = 0$
Derivative of a linear function	$f(x) = ax + b$	$\frac{d}{dx} f(x) = a$
Power rule	$f(x) = x^k$	$\frac{d}{dx} f(x) = kx^{k-1}$
Derivative of a constant multiple	$f(x) = ag(x)$	$\frac{d}{dx} f(x) = a \frac{d}{dx} g(x)$
Derivative of a sum	$f(x) = g(x) + h(x)$	$\frac{d}{dx} f(x) = \frac{d}{dx} g(x) + \frac{d}{dx} h(x)$
Product rule	$f(x) = g(x)h(x)$	$\frac{d}{dx} f(x) = g(x) \frac{d}{dx} h(x) + \frac{d}{dx} g(x) h(x)$
Quotient rule	$f(x) = \frac{g(x)}{h(x)}$	$\frac{d}{dx} f(x) = \frac{h(x) \frac{d}{dx} g(x) - \frac{d}{dx} h(x) g(x)}{[h(x)]^2}$
Chain rule	$f(x) = g(h(x))$	$\frac{d}{dx} f(x) = g'(h(x)) \cdot h'(x)$
Derivative of the natural log	$f(x) = \ln(x)$	$\frac{d}{dx} f(x) = \frac{1}{x}$
Derivative of the exponential	$f(x) = e^x$	$\frac{d}{dx} f(x) = e^x$

Notation: $\frac{d}{dx} f(x) = \frac{df}{dx} = f'(x)$

Calculus of Multiple Variables

- ▶ Function can have more than one variable: eg, $f(x, y) = x + y$
- ▶ When taking *partial derivative* of f with respect to x , keep y as a constant
- ▶ Notation:
 - ▶ $\frac{\partial}{\partial x}(f(x, y)) = \frac{\partial f}{\partial x} = f_x(x, y) = f_1(x, y)$
 - ▶ $\frac{\partial}{\partial y}(f(x, y)) = \frac{\partial f}{\partial y} = f_y(x, y) = f_2(x, y)$
- ▶ We use the notation $\frac{df}{dx}$ to indicate a *total* derivative, where we take into account any variables that vary with x
- ▶ Typically (but not always) we are interested in the partial derivative in this class

Total vs Partial Derivative

- ▶ For example, suppose $f(x, y) = xy$ but we know that $y = x^2$
 - ▶ What is partial derivative of f w.r.t x (as a function of y)?

$$\frac{\partial f}{\partial x} = y$$

- ▶ What is total derivative of f w.r.t x ?

$$\frac{df}{dx} = \frac{\partial}{\partial x} (x \cdot x^2) = \frac{\partial}{\partial x} (x^3) = 3x^2$$

- ▶ More generally, if $x = x(t)$ and $y = y(t)$, then

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$

by the chain rule and the product rule

Integration

- ▶ You should also know how to integrate functions of single and multiple variables as well
 - ▶ This is much less common than differentiation in this class
- ▶ To find the integral of a function $f(x)$, think about what function $F(x) + C$ would have derivative $f(x)$
 - ▶ This approach makes use of the fundamental theorem of calculus:

Theorem (FTOC)

Let f be a continuous function on the interval $[a, b]$. If F is the indefinite integral of f , then $\int_a^b f(x)dx = F(b) - F(a) \equiv F(x)|_{x=a}^{x=b}$.

Unconstrained Optimization: One Variable

- ▶ Suppose we are trying to find the optimum (ie maximum or minimum) of a function $f(x)$
- ▶ We use the *first order condition* to solve for the optimum:

$$\frac{d}{dx}f(x) = 0$$

- ▶ Value of x that satisfy the first order condition will be the values that optimizes $f(x)$
- ▶ How do we know if we have a maximum or a minimum?
 - ▶ We check the *second order conditions*:
 - ▶ If $\frac{d^2f}{dx^2} < 0$, we have a maximum
 - ▶ If $\frac{d^2f}{dx^2} > 0$, we have a minimum

Unconstrained Optimization: Multiple Variables

- ▶ Suppose we are trying to find the optimum of a function $f(x, y)$
- ▶ Again, we use the *first order conditions* to solve for the optimum, just now there are more than one:

$$\frac{\partial}{\partial x} (f(x, y)) = 0$$

$$\frac{\partial}{\partial y} (f(x, y)) = 0$$

- ▶ Values of x and y that satisfy the first order conditions will be the values that optimize $f(x, y)$

Constrained Optimization

- ▶ Suppose we are trying to find the optimum of the the function $f(x, y)$ subject to the constraint $b(x, y) \leq m$.
- ▶ First construct the *Lagrangian*:

$$\mathcal{L}(x, y, \lambda) = f(x, y) + \lambda(m - b(x, y))$$

- ▶ The variable λ is called the *Lagrange multiplier*
- ▶ Take the first order conditions of the Lagrangian:

$$\frac{\partial}{\partial x} (\mathcal{L}(x, y, \lambda)) = 0 \quad \frac{\partial}{\partial \lambda} (\mathcal{L}(x, y, \lambda)) = 0 \quad \frac{\partial}{\partial y} (\mathcal{L}(x, y, \lambda)) = 0$$

- ▶ Values of x and y that solve the FOC of the Lagrangian will be the same values that optimized the original function f subject to the constraint.