

Econ 301: Microeconomic Analysis

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Revealed Preference

Observing Choices

- ▶ So far in class: start with preferences/utility function, and derive choices
- ▶ Can we go the other way?
 - ▶ That is, can we derive preferences from observing choices?
- ▶ Two important assumptions throughout this section:
 1. Preferences are stable
 2. Strict convexity of preference (so unique maximum exists)

Revealed Preference

- ▶ Suppose we observe $X = (x_1, x_2)$ chosen when $Y = (y_1, y_2)$ was available at prices p_1, p_2

Definition

If X is chosen at prices p_1, p_2 and $p_1 y_1 + p_2 y_2 \leq p_1 x_1 + p_2 x_2$, then we say X is *directly revealed preferred* to Y .

- ▶ Often write this as $X \text{ DRP } Y$
- ▶ A better term: X is *chosen over* Y

Theorem

If $X \text{ DRP } Y$, then $X \succ Y$.

- ▶ That is, if we see X chosen over Y , we should be able to infer that $X \succ Y$

Chains of Revealed Preference

- ▶ What if we *also* observe $Y = (y_1, y_2)$ being chosen over $Z = (z_1, z_2)$ at different prices q_1, q_2 ?

Definition

If X is directly revealed preferred to Y and Y is directly revealed preferred to Z , then we say X is *indirectly revealed preferred* to Z , or X IRP Z .

- ▶ Note $X \succ Y$ and $Y \succ Z$ from theorem, which implies $X \succ Z$ by transitivity

Theorem

If X IRP Z , then $X \succ Z$.

Definition

If X DRP Z or X IRP Z , we say X is *revealed preferred* Z , or X RP Z .

Revealed Preference Graphically

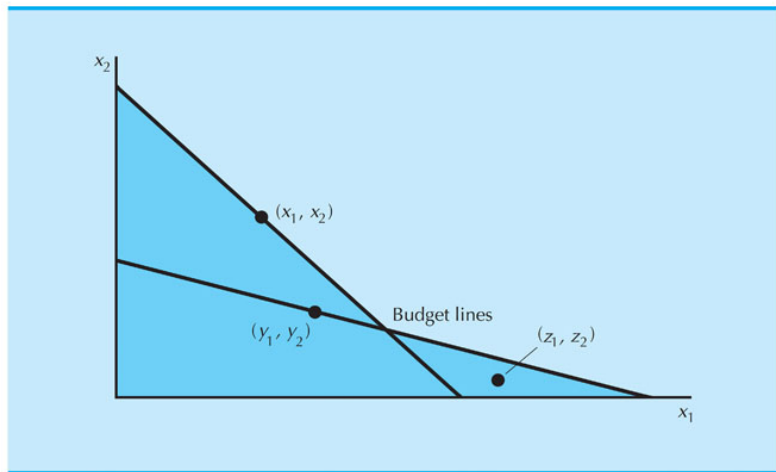


Figure
7.2

What This Buys Us

- ▶ What do we get from setting up all these definitions?
- ▶ Two really nice tricks to go from choices to preferences:
 1. We can put bounds on indifference curves by observing choices
 2. We can check whether observed choices are consistent with maximizing behavior

Recovering Preferences

- ▶ If we observe that X DRP S under budget 1 and S DRP T under budget 2, which bundles can we compare to X ?
 - ▶ $X \succ S \succ T$
 - ▶ Furthermore, X is strictly preferred to *everything* in budget set 1 and *everything* in budget set 2
- ▶ If we further observe Y DRP X and Z DRP X , which bundles can we compare to X ?
 - ▶ Any bundle with at least as much good 1 and good 2 as Y or Z is strictly preferred to X , by monotonicity
 - ▶ Furthermore, any bundle with at least as much good 1 and good 2 than any *mixture* of Y or Z with X is strictly preferred to X , by convexity and monotonicity
- ▶ In observing just 4 choices, we have put strong restrictions on where indifference curve through X can lie

Recovering Preferences Graphically

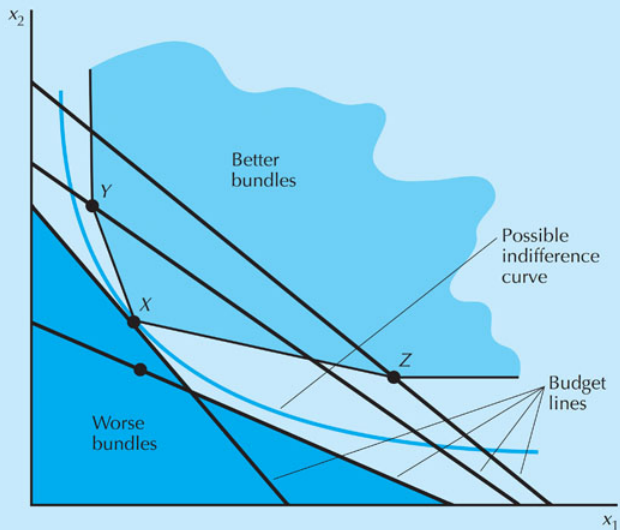


Figure
7.3

Testing Choices: The Weak Axiom

- ▶ Suppose we observe X DRP Y from one budget set and Y DRP X from another budget set
 - ▶ Implies $X \succ Y$ and $Y \succ X$, a contradiction
 - ▶ Clearly such data would imply that consumer is not maximizing

Definition

The *Weak Axiom of Revealed Preference (WARP)* states that if X is directly revealed preferred to Y , then we cannot have Y directly revealed preferred to X .

Theorem

If WARP is violated, then we can conclude that observed behavior is not consistent with maximizing model of consumer choice

Testing WARP

- ▶ Given data on prices and bundles chosen at those prices, how to we check for WARP violations?
- ▶ We use the following algorithm:
 - ▶ Calculate expenditure for each bundle at each possible price
 - ▶ Expenditure is $p_1 x_1 + p_2 x_2$
 - ▶ Put cost of bundle i at price point j in row i , column j in matrix, ie cell (i, j)
 - ▶ Direct revealed preference when an off-diagonal entry is cheaper than then on-diagonal entry in the same column
 - ▶ Indicate direct revealed preference with *
 - ▶ If * in at cell (r, c) and (c, r) , violation of WARP

Testing WARP: Example

- Suppose we observe the following three choices under the given prices:

Choice Number	p_1	p_2	x_1	x_2
1	1	2	1	2
2	2	1	2	1
3	1	1	2	2

- Is there a violation of WARP?

Choice Number	Budget 1	Budget 2	Budget 3
1	5	4*	3*
2	4*	5	3*
3	6	6	4

- Violation: choice 1 DRP choice 2 and choice 2 DRP choice 1

Testing Choices: The Strong Axiom

- ▶ The weak axiom works only one way:
 - ▶ If we have a violation, then consumer is not maximizing
 - ▶ But if we don't find a violation, we can't be sure if consumer is maximizing
 - ▶ That is, satisfying the weak axiom is necessary but not sufficient for maximizing behavior
- ▶ Luckily we have another condition that is both necessary and sufficient for maximizing behavior:

Definition

The *Strong Axiom of Revealed Preference (SARP)* states that if X is revealed preferred (directly or indirectly) to Y , then Y cannot be revealed preferred to X (directly or indirectly).

Index Numbers

Quantity Indices

- ▶ Suppose we want to compare average purchasing behavior today (period t) to average purchasing behavior in some baseline year (period b)
- ▶ Needs weights w_1 and w_2 on goods 1 and 2:

$$I_q = \frac{w_1 x_1^t + w_2 x_2^t}{w_1 x_1^b + w_2 x_2^b}$$

- ▶ Weights tell us relative importance of the two good when evaluations how well off consumer is

Paasche quantity index

- ▶ Let weights be today's prices
- ▶ This is called *Paasche quantity index*

$$P_q = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^t x_1^b + p_2^t x_2^b}$$

- ▶ If $P_q > 1$, what can we say about how well off consumer is today vs baseline?
 - ▶ Note X^t DRP X^b , so consumer is better off today
- ▶ If $P_q < 1$, what can we say about how well off consumer is today vs baseline?
 - ▶ Can't say definitively

Laspeyres quantity index

- ▶ Let weights be baseline year's prices
- ▶ This is called *Laspeyres quantity index*

$$L_q = \frac{p_1^b x_1^t + p_2^b x_2^t}{p_1^b x_1^b + p_2^b x_2^b}$$

- ▶ If $L_q < 1$, what can we say?
 - ▶ Note X^b DRP X^t , so consumer is better off in baseline
- ▶ If $L_q > 1$, what can we say?
 - ▶ Can't say definitively

Price Indices

- ▶ Quantity indices compare average consumption in two periods
- ▶ What if we want to compare average prices?
 - ▶ Use price indices:

$$I_p = \frac{p_1^t w_1 + p_2^t w_2}{p_1^b w_1 + p_2^b w_2}$$

Paasche Price Index

- ▶ Use today's consumption as weight:

$$P_p = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^t + p_2^b x_2^t}$$

- ▶ What can we say if $P_p < 1$?
 - ▶ Nothing: need to consider how income changes
- ▶ Define relative change in income:

$$M = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^b + p_2^b x_2^b}$$

- ▶ If $P_p > M$, what can we say?
 - ▶ Consumer is better off in base year because X_b DRP X_t
 - ▶ Intuition: Income needs to increase faster than price changes for consumer to be *definitely* better off today
- ▶ If $P_p < M$, what can we say?
 - ▶ Can't say whether consumer is better off now or at baseline

Laspeyres Price Index

- ▶ Use baseline period's consumption as weight:

$$L_p = \frac{p_1^t x_1^b + p_2^t x_2^b}{p_1^b x_1^b + p_2^b x_2^b}$$

- ▶ If $L_p < M$, what can we say?
 - ▶ Consumer is better off today because X_t DRP X_b
- ▶ If $L_p > M$, what can we say?
 - ▶ Can't say