

Econ 301: Microeconomic Analysis

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The Slutsky Equation

Total Change in Demand

- ▶ Last time we defined
 - ▶ Income effect: $\Delta x_1^n = x_1(p'_1, m) - x_1(p'_1, m')$
 - ▶ Substitution effect: $\Delta x_1^s = x_1(p'_1, m') - x_1(p_1, m)$
- ▶ Note that the total change in demand is

$$\begin{aligned}\Delta x_1 &= x_1(p'_1, m) - x_1(p_1, m) \\ &= x_1(p'_1, m') - x_1(p_1, m) + x_1(p'_1, m) - x_1(p'_1, m') \\ &= \Delta x_1^s + \Delta x_1^n\end{aligned}$$

- ▶ This is one form of the *Slutsky identity* or *Slutsky equation*

Signing the Total Change in Demand

- ▶ What sign does the overall change in demand take if $\Delta p_1 < 0$?
- ▶ We showed that $\Delta x_1^s > 0$ if $\Delta p_1 < 0$
- ▶ What if good 1 is normal?
 - ▶ Then $\Delta x_1^n > 0$, so $\Delta x_1 > 0$
- ▶ What if good 1 is inferior?
 - ▶ Then $\Delta x_1^n < 0$, so sign of Δx_1 is ambiguous
 - ▶ Note $\Delta x_1 < 0$ (a Giffen good) if $|\Delta x_1^n| > |\Delta x_1^s|$
- ▶ Are all Giffen goods inferior? Yes
- ▶ Are all inferior goods Giffen? No

Giffen and Inferior Goods Graphically

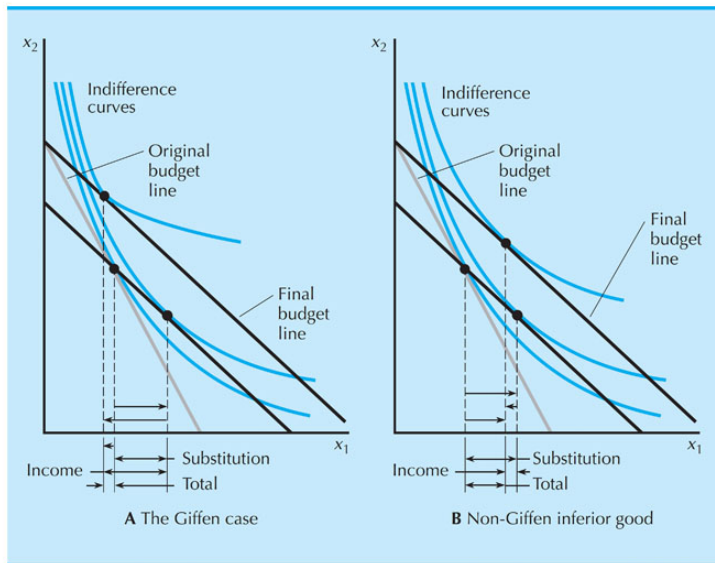


Figure
8.3

Example

- ▶ Let demand function be given by $x_1 = 10 + \frac{m}{10p_1}$
- ▶ Suppose we start out at $p_1 = 3$ and $m = 120$
- ▶ Suppose price decrease to $p'_1 = 2$
- ▶ Substitution effect?
 - ▶ Starting demand $x_1 = 10 + \frac{120}{30} = 14$
 - ▶ $\Delta m = x_1 \Delta p_1 = 14(-1) = -14$
 - ▶ $m' = 120 - 14 = 106$
 - ▶ Intermediate demand $x_1(p'_1, m') = 10 + \frac{106}{20} = 15.3$
 - ▶ $\Delta x_1^s = x_1(p'_1, m') - x_1(p_1, m) = 15.3 - 14 = 1.3$
- ▶ Income effect?
 - ▶ Final demand $x_1(p'_1, m) = 10 + \frac{120}{20} = 16$
 - ▶ $\Delta x_1^i = x_1(p'_1, m) - x_1(p'_1, m') = 16 - 15.3 = 0.7$

Perfect Complements

- ▶ Consider perfect complements preferences
- ▶ What are the income and substitution effects of a decrease in p_1 ?
 - ▶ Note that pivot has no effect on optimal consumption
 - ▶ Thus substitution effect is zero
 - ▶ Thus demand change is entirely from income effect

Perfect Complements Graphically

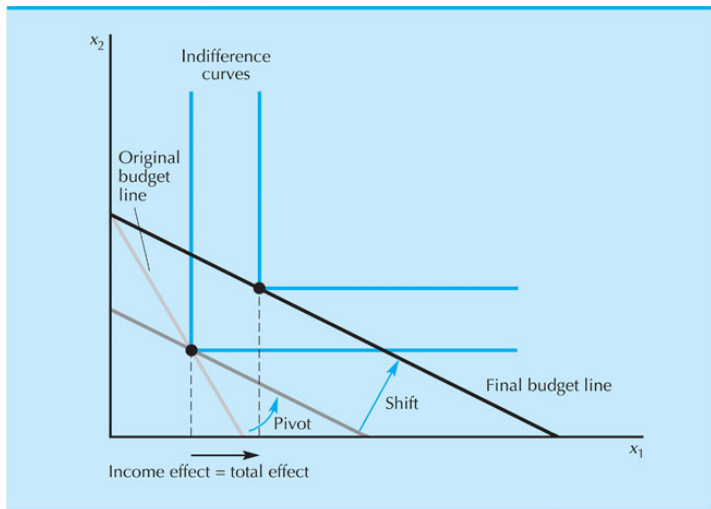


Figure 8.4

Perfect Substitutes

- ▶ Consider perfect substitutes (starting where all consumption is of good 2)
- ▶ What are the income and substitution effects of a decrease in p_1 ?
 - ▶ Note that after pivot, no shift is required to get back to original income level
 - ▶ Thus income effect is zero
 - ▶ Thus all of demand change (if any) is driven by substitution effect
 - ▶ If change in price is small enough, substitution effect will be zero too

Perfect Substitutes Graphically

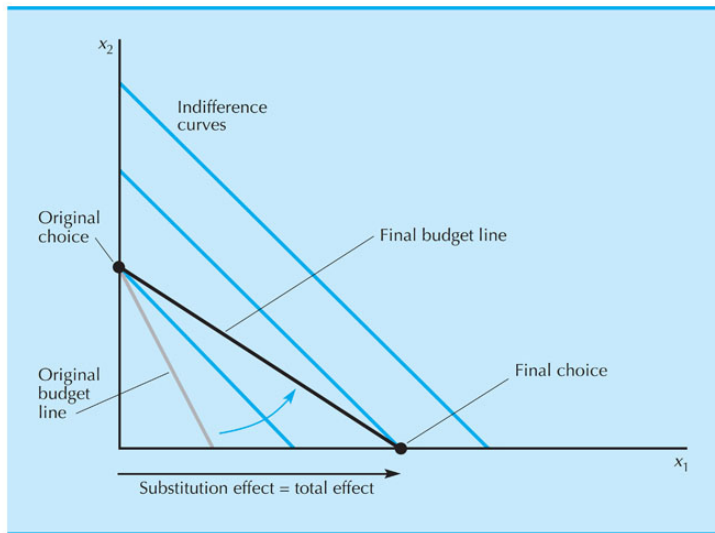


Figure
8.5

Quasilinear Preferences

- ▶ Quasilinear preferences (quasilinear in good 2):

$$u(x_1, x_2) = v(x_1) + x_2$$

- ▶ Note that MRS depends only on x_1 :

$$MRS = -\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = -v'(x_1)$$

- ▶ Thus the indifference curves are vertical translations of one another
- ▶ Substitution effect: pivot moves consumer to higher indifference curve
- ▶ Income effect: shift moves consumer to tangency point directly above, hence effect is zero

Quasilinear Preferences Graphically

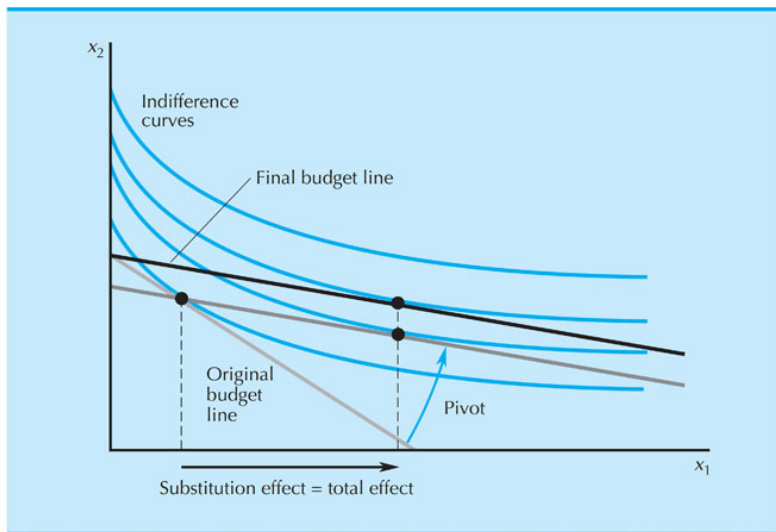


Figure
8.6

Rates of Change

- ▶ We can make a second formulation of the Slutsky equation
- ▶ First, define the negative income effect as $\Delta x_1^m = -\Delta x_1^n$
- ▶ Then the Slutsky equation is

$$\Delta x_1 = \Delta x_1^s - \Delta x_1^m$$

- ▶ Divide through by Δp_1 :

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta p_1}$$

- ▶ Finally, substitute $\Delta p_1 = \frac{\Delta m}{x_1}$ into rightmost term:

$$\underbrace{\frac{\Delta x_1}{\Delta p_1}}_{\text{total effect}} = \underbrace{\frac{\Delta x_1^s}{\Delta p_1}}_{\text{sub effect}} - \underbrace{\frac{\Delta x_1^m}{\Delta m}}_{\text{inc effect}} x_1$$

Confirming with Example

- ▶ Consider our example from earlier:
 - ▶ Demand function $x_1 = 10 + \frac{m}{10p_1}$
 - ▶ $p_1 = 3$ and $m = 120$
 - ▶ Price decrease to $p'_1 = 2$
- ▶ Does the rates of change version of Slutsky hold?
- ▶ We found:
 - ▶ $x_1 = 14$
 - ▶ $\Delta m = -14$
 - ▶ $\Delta x_1^s = 1.3$
 - ▶ $\Delta x_1^n = 0.7$
- ▶ Confirm Slutsky:
 - ▶ $\frac{\Delta x_1}{\Delta p_1} = \frac{2}{-1} = -2$
 - ▶ $\frac{\Delta x_1^s}{\Delta p_1} + \frac{\Delta x_1^m}{\Delta m} x_1 = \frac{1.3}{-1} - \frac{-0.7}{-14} 14 = -1.3 - 0.7 = -2$

Slutsky with Calculus

- ▶ We can get a third and final version of Slutsky from calculus principles
- ▶ First, define the *Slutsky demand* function as

$$x_1^s(p_1, p_2, \bar{x}_1, \bar{x}_2) = x_1(p_1, p_2, \underbrace{p_1 \bar{x}_1 + p_2 \bar{x}_2}_m)$$

where (\bar{x}_1, \bar{x}_2) is original demand bundle

- ▶ Differentiating:

$$\frac{\partial x_1^s}{\partial p_1} = \frac{\partial x_1}{\partial p_1} + \frac{\partial x_1}{\partial m} \frac{\partial m}{\partial p_1}$$

- ▶ Finally, noting that $\frac{\partial m}{\partial p_1} = \bar{x}_1$ and rearranging:

$$\frac{\partial x_1}{\partial p_1} = \frac{\partial x_1^s}{\partial p_1} - \frac{\partial x_1}{\partial m} \bar{x}_1$$

Confirming with Example

- ▶ Consider our example demand function $x_1 = 10 + \frac{m}{10p_1}$
- ▶ Confirm Slutsky:
 - ▶ $\frac{\partial x_1}{\partial p_1} = -\frac{m}{10p_1^2}$
 - ▶ Note that $x_1^s = 10 + \frac{p_1 x_1 + p_2 x_2}{10p_1} = 10 + \frac{x_1}{10} + \frac{p_2 x_2}{10p_1}$
 - ▶ So $\frac{\partial x_1^s}{\partial p_1} = -\frac{p_2 x_2}{10p_1^2}$
 - ▶ Note $p_2 x_2 = m - p_1 x_1$, so $\frac{\partial x_1^s}{\partial p_1} = -\frac{m - p_1 x_1}{10p_1^2} = -\frac{m}{10p_1^2} + \frac{x_1}{10p_1}$
 - ▶ $\frac{\partial x_1}{\partial m} = \frac{1}{10p_1}$
 - ▶ Thus $\frac{\partial x_1^s}{\partial p_1} - \frac{\partial x_1}{\partial m} x_1 = -\frac{m}{10p_1^2} - \frac{x_1}{10p_1} + \frac{1}{10p_1} x_1 = -\frac{m}{10p_1^2}$

Appendix

Compensated Demand

- ▶ We can decompose demand change in response to price change in another way
- ▶ First, “roll” budget curve along indifference curve until get to new budget curve slope
 - ▶ This is called *Hicksian demand* or *compensated demand*
 - ▶ Note that we keep utility the same during first move
- ▶ Then, shift demand out by increasing income

Compensated Demand Decomposition Graphically

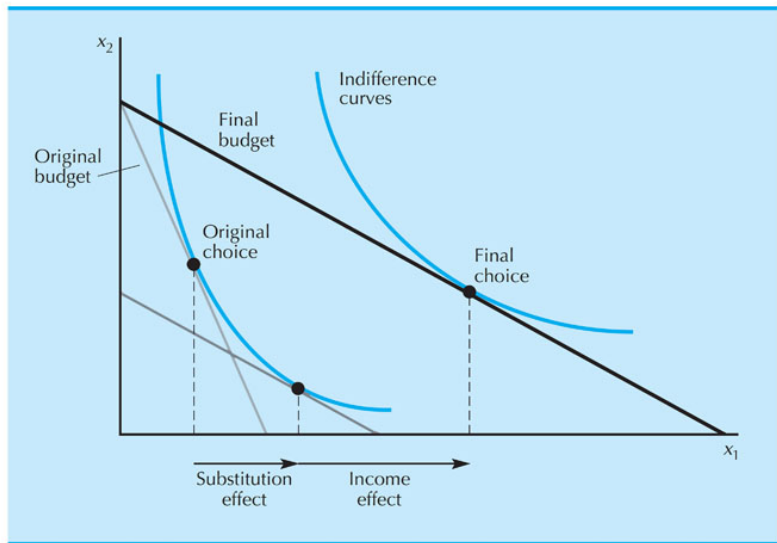


Figure
8.9

Sign of Compensated Demand Substitution Effect

- ▶ What sign does compensated demand substitution effect (ie “roll”) take?
- ▶ Note that $(x_1, x_2) \sim (y_1, y_2)$, so we must have

$$p_1 x_1 + p_2 x_2 \leq p_1 y_1 + p_2 y_2$$

$$p'_1 y_1 + p_2 y_2 \leq p'_1 x_1 + p_2 x_2$$

- ▶ Adding these equations together:

$$(p'_1 - p_1)(y_1 - x_1) + (p_2 - p_2)(y_2 - x_2) \leq 0$$

- ▶ Since second term is zero we get

$$\Delta p_1 \Delta x_1 \leq 0$$

- ▶ Thus a decrease in p_1 causes an increase in compensated demand (just like with Slutsky)

Different Demands

- ▶ In fact, have a Slutsky-like decomposition using compensated demand:

$$\frac{\partial x_1}{\partial p_1} = \frac{\partial x_1^c}{\partial p_1} - \frac{\partial x_1}{\partial m} x_1$$

where $x_1^c(p_1, p_2, \bar{u})$ is compensated demand for a particular utility level \bar{u}

- ▶ If we want to see how demand changes with price changing and ...
 - ▶ income fixed: use standard demand (also called *Marshallian demand*)
 - ▶ purchasing power fixed: use Slutsky demand
 - ▶ utility fixed: use Hicksian/compensated demand